GENERAL RELATIVITY applied to a

CHARGE COUPLE

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I. Introduction

The scenario is simple. Two "naked", (massless) charges "a" and "b" are at rest separated by some distance, "S" (the charges are fundamental = 4.803×10^{-10} esu). Neither charge alone possesses mass-energy, but the system has mass-energy (c = 1), $m = a \cdot b/S$ from classical EM theory. The distance "S" is measured using a light signal, like radar ranging.

To solve the metric "g_{uv}" in this case, one cannot use $G_{uv} = 0$, as is traditional in GR, due to the inter-dependence of the charges that create the system mass "m", so $G_{uv} = T_{uv}$ is employed (the constant $-8\pi G$ is suppressed into "T"). Because the situation is simplified to static, solving $G_{00} = T_{00}$ will provide sufficient conceptual accuracy. Reference to Weinberg's, "Gravitation and Cosmology" Eq.(7.1.3) and Eq.(7.1.7).

$$G_{00} = \nabla^2 (g_{00}) = T_{00} \tag{1}$$

The solution I obtain is as follows,

$$g_{00} = 1 - (a/S) \cdot (b/S) = 1 - A \cdot B \tag{2}$$

with "A" and "B" being potentials. The $\nabla^2 A$ = Laplacian (A) = 0, and likewise for "B". Classically the Electric field $E(a) = \nabla A$ (+/- won't make a diff, here). So we collect that up and find, Eq.(1) yields,

$$G_{00} = E(a) \cdot E(b) = T_{00} . \tag{3}$$

and letting $E(a) \cdot E(b) = (a/S^2) \cdot (b/S^2)$ gives,

$$T_{00} = m/S^3 \; , \; (m = a \cdot b/S) \, ,$$

the energy density.

We can apply the metric Eq.(2) to find,

$$S^2 = X^2 + a \cdot b \, . \tag{4}$$

Defining better the meaning: In a *flat* orthogonal field (3D space is fine) without any masses or charges we imagine setting two Points P(a) and P(b). The distance between P(a) and P(b) in the absence of any field that may change the velocity of light using the above definition of orthogonal space is X. That's why I call that imaginary, because you really need hard objects to bounce photons from to do a real survey.

When charges are assigned to the locations P(a) and P(b) we'll define the distance "S" between them by using light waves, but because the "energy density" of the field is altered by Eq.(3), the velocity of light will not be constant, and so the measure of X and S will be different, as defined in Eq.(4). For example, let $q = |a| = |b|$ be our generic fundamental charge, with a positive magnitude, then, using Eq.(4),

$$S^2 = X^2 + q^2 \quad \text{(repelling charges, } a = b\text{),}$$

$$S^2 = X^2 - q^2 \quad \text{(attracting charges, } a = -b\text{).}$$

We see S(repel) > S(attract), and so Attraction > Repulsion that difference is in accord with gravitational force, and in accord with GR, where the solution originated.

For example: One can see how gravitational force arises from Coulomb's force using the approximation,

$$F = a \cdot b/S^2 = a \cdot b/(X^2 + a \cdot b) \sim a \cdot b/X^2 - (a \cdot b)^2 / X^4 \qquad (5)$$

The last term in Eq.(5) is always negative, independent of the relative polarities of charges "a" and "b", hence it is a residual attractive force in the charge couple. Letting $M = a \cdot b/X$ be a mass-energy, that term becomes,

$$f = - M \cdot M/X^2 ,$$

which is basically Newtonian gravitational force.

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