Modern Space-Time

Ken S. Tucker

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Abstract

$\underline{The\ Interdefinition\ of\ Length\ and\ Time.}$

Since 1983, the meter, second and speed of light "c" have been defined by N meters = c ∗ 1 second with N = 299,792,458.
We find, that's had a profound effect on Relativity Theory.
In the MKS system of units we have the basis vectors $\vec{e}^0$, $\vec{e}^1$, $\vec{e}^2$, $\vec{e}^3$ such that the magnitudes are

$$|\vec{e}^i| = 1\,meter,\ \{i = 1, 2, 3\}\ and\ |\vec{e}^0| = c * 1\ second. \tag{1}$$

These are related by $N * |\vec{e}^i| = |\vec{e}^0|$ as per International definition. We may of course eliminate the N by an arbituary change in the length scale to simplify to $|\vec{e}^i| = |\vec{e}^0|$.

With it understood the $\vec{e}^\mu$ are the 4D basis vectors, we define our metrics by the scalar products herein,

$$g_{uv} = \vec{e}_u \cdot \vec{e}_v\ and\ g^{uv} = \vec{e}^u \cdot \vec{e}^v, \tag{2}$$

being the covariant and contravariant metric tensors.

In a Cartesian Coordinate System that yields,

$$g_{00} = g_{11} = g_{22} = g_{33} = 1. \tag{3}$$

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$\underline{An\ Alternative\ to\ the\ conventional\ Minkowski\ Metric}$.
In flat space you set $g_{00} = g_{11} = g_{22} = g_{33} = 1$ and $g_{0i} = -dx^i/dx^0$, that works very well when merging SR with GR.
The thing most researchers avoid are nonorthogonal metrics like g_{0i}, but aberration happens when relative speed separates CS's, so you can't really hide it by using $\sqrt{(}-1)$, just because g_{0i} looks complicated.
Unfortunately SR was established algebraically, and a sort of evolution occurred employing the use of tensors by Einstein for GR. What should have happened, in a more ideal historical hindsight, is to begin with,

$$U_\mu U^\mu = 1\ ,\ \{\mu = 0, 1, 2, 3\}\ ,\ U^\mu = dx^\mu/ds \tag{4}$$

as a definition in SR. Then expand that to detail time and space as,

$$U_0U^0 + U_iU^i = 1, \ \{i = 1, 2, 3\} \tag{5}$$

The U_iU^i is *absolute velocity* and since one can always find a CS where motion of something is zero, is the same as saying motion is relative, hence, $U_i \ U^i \ = \ 0 = absolute\ motion$ is the covariant way (for all CS's using tensors) of writing "motion is relative".
Of course relative motion is retained by U^i (by choice) and being non-zero generally produces,

$$U_i = 0, \ generally. \tag{6}$$

Now you can use association to obtain,

$$U_i = g_{i\mu}U^\mu = 0 \tag{7}$$

and expand index "μ" in time and space $\{0, i\}$ to,

$$0 = g_{i0}U^0 + g_{ij}U^j. \tag{8}$$

Use a bit of algebra and see,

$$g_{i0} = -g_{ij}U^j/U^0 = -g_{ij}dx^j/dx^0. \tag{9}$$

Specifing a flat space-time metric g_{ij} simplifies to the Kronecker delta and so, $g_{i0} \ = \ -dx^i/cdt$ simplified, and is aberration...a real effect well established by experiment.
Now let's stick those nonorthogonal critters in $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$
by expanding indices "μ" and "ν" over time and space,

$$ds^2 \ = \ g_{00}dx^0dx^0 + 2g_{0i}dx^0dx^i + g_{ij}dx^idx^j \tag{10}$$

From far above, sub in $g_{0i} = -g_{ij}dx^j/dx^0$ and get

$$ds^2 = g_{00}dx^0dx^0 - g_{ij}dx^idx^j \ (generally). \tag{11}$$

Sub in a simplified metric $g_{00}, g_{11}... = 1 \ and \ dx^0 = cdt$ to get the familiar

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 \tag{12}$$

that Minkowski and later Einstein needed for GR.
In relativity, the important thing is to deal with relative motion and the vanishing of *absolute* motion at the outset, otherwise a lot of spooky-kooky aberrations appear, (I won't hang other's dirty laundry in public), that I think is best expressed by the tidy, $U_i = 0$.
Anyway if the metric system you choose finds $U_i \ U^i \ \neq \ 0$
you may have a problem.
Above, we see the only metric compatible with $U_i = 0$ is,

$$ds^2 \ = \ g_{00}dx^0dx^0 - g_{ij}dx^idx^j \ (generally). \tag{13}$$

Note the total absence of g_{i0} terms when absolute motion is excluded and relativity is strictly invoked, essentially "collapsing" the metric relativistically.
Finally, we need to ask if a non-zero result of the equation, U_iU^i is truly indicative of absolute motion as I've assumed?

Regards
Ken S. Tucker